SHORTER ARTICLES AND DISCUSSION

ON THE NUMERICAL EXPRESSION OF THE DEGREE OF INBREEDING AND RELATIONSHIP IN A PEDIGREE[1]

Dr. Raymond Pearl has given in a series of papers ("Studies on Inbreeding," I–VIII) published in the American Naturalist during the years 1913 to 1917 a system of measuring numerically the degree of inbreeding and relationship in a pedigree.

It will not be necessary to describe the method in any length as it may be familiar to most readers or can easily be found in the original papers.

The starting point is the fact, that all inbred individuals, notwithstanding the special system of inbreeding involved, have fewer different ancestors in a certain generation than the greatest possible number.

The degree of inbreeding is measured by the extent of this reduction in numbers of different ancestors. For that purpose a *coefficient of inbreeding* is determined for each generation according to the formula

$$Z_n = \frac{100(p_{n+1} - q_{n+1})}{p_{n+1}},$$

where p_{n+1} indicates the greatest possible number of different ancestors in the $n + 1$st generation and q_{n+1} means the actual number of different ancestors in the same generation.

Plotting the series of values obtained for Z over a base indicating the series of generations, the *inbreeding curve* can be drawn. Maximum values for the coefficient of inbreeding are obtained when continuous brother and sister mating is involved. The brother and sister inbreeding curve has therefore the importance as the limit, which no other inbreeding curve can surpass.

The proportion of the actual inbreeding during a number of generations to the highest possible inbreeding in the same number of generations then offers a fairly good measure for the

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total inbreeding. This proportion can be found by determining the proportion of the area included by the actual inbreeding curve in percentage of the area included by the maximum inbreeding curve. The area of the latter can be calculated by integration. The actual curve is usually so irregular that no integration is possible. To get an approximate value the series of values for Z is simply summed. The formula for the *total inbreeding coefficient* is then the following:

$$Z_{T_n} = \frac{100\Sigma \frac{Zn}{Zl}}{F_{T_n}},$$

the Σ denoting the summation of all values between and including the limits indicated, and F_{T_n} is the area included by the maximum curve, the same number of generations being involved.[2]

A question of considerable genetic bearing is the degree of relationship between the parents of the individual, whose pedigree is being examined. Relationship is indicated by the reappearance of individuals from the pedigree of one individual in the ancestry of another.

As a measure of degrees of relationship Pearl proposed a *coefficient of relationship*. This is calculated in two slightly different ways according to whether it is measured with the coefficient of inbreeding, where the relationship of the sire and dam of the inbred individual is being calculated, or separately, when relationship of any two individuals is measured. The formulæ are as follows:

$$(1) \qquad K_n = \frac{100(p_{n+1} - q_{n+1}) - ({}_sZ_{n-1}\cdot {}_sp_n + {}_dZ_{n-1}\cdot {}_dp_n)}{1/2p_{n+1}},$$

$$(2) \qquad K_n = \frac{100(p_{n+1} - r_{n+1})}{1/2p_{n+1}},$$

where a prefixed s or d indicates that the following letter refers to the sire's or the dam's pedigree only, and $(p_{n+1} - r_{n+1})$ means the number of common ancestors for both pedigrees in the $n + 1$st generation.

As a measure of the proportion of the actual inbreeding due to relationship between the sire and dam of the individual Pearl has proposed to give a series of *partial inbreeding indices* calculated according to the formula

$$KZ_n = \frac{50(K_n)}{Z_n}.$$

[2] The values of F_{T_n} are given in Table I, "Studies on Inbreeding," VIII, 1917 (Pearl).

In this paper I wish to give a modification and some extensions to the method worked out by Pearl with the purpose of bringing all the measurements of inbreeding and relationship on the same scale and using total coefficients based on calculations of areas as the fundamental method in expressing degrees of kinship numerically.

I. Definition of the Coefficient of Relationship

If we plot the values of the coefficient of relationship obtained in accordance with the above indicated method together with the values of the corresponding coefficients of inbreeding for a given pedigree, we shall find that these corresponding values are not directly comparable as they are not worked out in relation to the same scale. To obtain this we need to change slightly the definition and formula of the coefficient of relationship.

Pearl's definition is the following: The coefficient of relationship indicates the number of ancestors common to both pedigrees of the two individuals whose relationship is being measured in proportion to the greatest possible number of *common* ancestors in this generation.

The proposed definition is this: The coefficient of relationship indicates the number of ancestors common to both pedigrees of the two individuals whose relationship is being measured in proportion to the total maximum number of *different* ancestors in the two pedigress taken together in the generation in question.

The formulæ are now the following:

(1) (When the relationship between the sire and dam of an inbred individual is being measured)

$$K_n = \frac{100(p_{n+1} - q_{n+1}) - (sZ_{n-1} \cdot sp_n + dZ_{n-1} \cdot dp_n)}{p_{n+1}};$$

(2) (If any two different individuals are concerned)

$$K_n = \frac{100(p_{n+1} - r_{n+1})}{p_{n+1}}.$$

The difference between these formulæ and the first mentioned is only that the denominator in the fractions is multiplied by two. The total values, therefore, are exactly one half of Pearl's.

The maximum value of the coefficient of relationship will in every generation be 50, as no more than 50 per cent. of the individuals in a generation of a pedigree can appear in both halves of the pedigree.

II. The Total Relationship Coefficient

To measure the total degree of relationship during a number of generations I propose to use a total relationship coefficient based on the same common principles as the total inbreeding coefficient.

The areas to be compared are the area included by the relationship curve (corresponding to the inbreeding curve) and the area included by the maximum relationship curve. Now the maximum values of the coefficient of relationship are 50 in every generation beginning at K_1 and the area in question is for that reason simply 50 times the number of generations involved.

The formula of the total relationship coefficient for n generations is then the following:

$$K_{Tn} = \frac{100\Sigma_{K1}^{Kn}}{50n} = \frac{2\Sigma_{K1}^{Kn}}{n}.$$

III. Total Relationship Inbreeding Index

To indicate the proportion of the inbreeding that is due to relationship between the sire and dam of the individual, whose pedigree is being examined, I wish to propose a single numerical expression, the total relationship inbreeding index indicating the proportion of the area included by the relationship curve to the area included by the inbreeding curve, the formula being the following:

$$KZ_{Tn} = \frac{100\Sigma\frac{Kn}{Kl}}{\Sigma\frac{Zn}{Zl}}.$$

Calculation of the Coefficient Indicating the Degree of Inbreeding and Relationship in the Pedigree of the Jersey Bull King Melia Rioter 14th (103901)

To show the calculation and significance of the described coefficients I have selected the pedigree of King Melia Rioter Fourteenth including eleven generations. Table I gives the

TABLE I

$Z_1 = 25.00$	$K_1 = 25.00$
$Z_2 = 25.00$	$K_2 = 25.00$
$Z_3 = 37.50$	$K_3 = 31.25$
$Z_4 = 50.00$	$K_4 = 37.50$
$Z_5 = 71.88$	$K_5 = 43.75$

$Z_6 = 81.25$	$K_6 = 46.09$
$Z_7 = 90.63$	$K_7 = 46.48$
$Z_8 = 92.77$	$K_8 = 46.88$
$Z_9 = 93.65$	$K_9 = 46.88$
$Z_{10} = 93.85$	$K_{10} = 46.88$
Total 661.53	Total 395.71

values of the series of the coefficients of inbreeding and relationship for each generation.

The calculations of the total coefficients are now the following:

$$\mathrm{Z}_{\mathrm{T}_{10}} = \frac{100 \times 661.53}{900.10} = 73.50,$$

$$\mathrm{K}_{\mathrm{T}_{10}} = \frac{2 \times 395.71}{10} = 79.14,$$

$$\mathrm{KZ}_{\mathrm{T}_{10}} = \frac{100 \times 395.71}{661.53} = 59.82,$$

Or expressed verbally: In eleven generations King Melia Rioter Fourteenth is 73.50 per cent. inbred, his sire and dam are 79.14

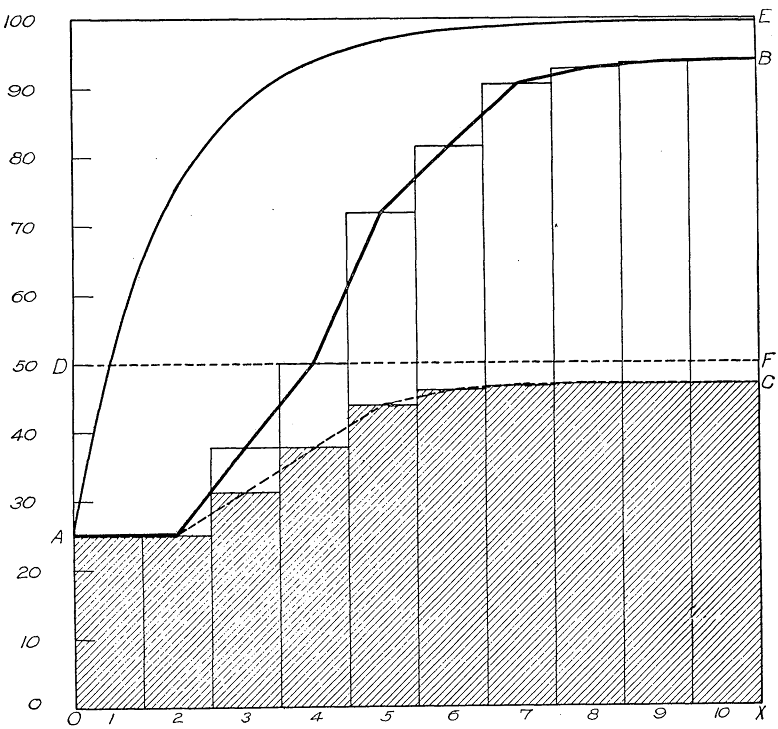

FIG. 1. For explanation see text.

per cent, related, and the part of the inbreeding due to relationship between his sire and dam is 59.82 per cent. of the actual total inbreeding.

In Fig. 1 the inbreeding curve and the relationship curve are plotted, based on the figures given in Table I, the former as a solid line, the latter as a broken line. The smooth curve indicates the maximum inbreeding curve; the broken line, that divides the area in two equal halves, indicates the maximum relationship curve. These four curves taken together give a fairly good graphical demonstration of the facts in question.

1. The area OABX in relation to the area OAEX gives the proportion of the actual to the maximum degree of inbreeding: *The total inbreeding coefficient.*

2. The area OACX in relation to the area ODFX indicates the proportion of the actual to the maximum degree of relationship: *The total relationship coefficient.*

3. The area OACX in relation to the area OABX gives the proportion of the inbreeding that is due to relationship: *The total relationship inbreeding index.*

In bringing all measurements of degrees of inbreeding and relationship to the same scale and using areas as the measures we get a uniform and significant series of coefficients that numerically express the degree of inbreeding and relationship in a given pedigree.

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SOME OBSERVATIONS CONCERNING THE PERIODICAL CICADA

DURING the recent visitation of the periodical cicada, their great abundance on the writer's home grounds at Vinson Station, Va., afforded an excellent opportunity to observe some of the habits of these interesting insects. During the months of January, February and March, the writer was engaged in clearing off all trees and brush from several lots immediately adjoining his home grounds. In the course of this work, several large oak trees were completely dug up by the roots. Even during the winter months, many of these benumbed creatures were encountered in their burrows in the soil around the roots. As warmer weather approached, their burrows became more numerous in the soil and it was evident that they were approaching the warmer, uppermost layer in ever-increasing numbers. Finally,